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REFLECTIONS ON ETHNOMODELLING AS AN INTERCULTURAL CONCEPT ON ETHNOMATHEMATICS AND MATHEMATICAL MODELLING

REFLEXÕES SOBRE A ETNOMODELAGEM COMO UM CONCEITO INTERCULTURAL EM ETNOMATEMÁTICA E MODELAGEM MATEMÁTICA

REFLEXIONES SOBRE LA ETNOMODELACIÓN COMO UN CONCEPTO INTERCULTURAL SOBRE LA ETNOMATEMÁTICA Y LA MODELIZACIÓN MATEMÁTICA

Daniel Clark Orey[1*]   Milton Rosa[2**]

ABSTRACT

The application of techniques in ethnomathematics along with the tools of modelling allows us to see a different reality and gives us insight into a holistic mathematics. In this perspective, the pedagogical approach that connects cultural aspects of mathematics with its school and/or academic aspects is named ethnomodelling, which is a process of translation and elaboration of problems and questions taken from systems that are part of the reality for any given cultural group. In this article we have offered an alternative goal for research, which is the acquisition of both emic and etic knowledge for the implementation of ethnomodelling. Emic knowledge is essential for an intuitive and empathic understanding of mathematical ideas of a culture, and it is essential for conducting effective ethnographic fieldwork through a dialogical approach of ethnomodelling.

Keywords: Ethnomodelling, Etnomathematics, Mathematical Modelling, Ethnomodels, Dialogical Approach.

RESUMO

A aplicação de técnicas etnomatemáticas juntamente com as ferramentas de modelagem nos possibilita ver uma realidade diferente e nos dá informações sobre uma matemática holística. Nessa perspectiva, a abordagem pedagógica que conecta aspectos culturais da matemática com os seus aspectos escolares e/ou acadêmicos é denominada de etnomodelagem, que é um processo de tradução e elaboração de problemas e questões presentes nos sistemas que são parte da realidade de qualquer grupo cultural. Neste artigo, oferecemos um objetivo alternativo para a pesquisa, que é a aquisição de conhecimento êmico e ético para a implementação da etnomodelagem. O conhecimento êmico é essencial para uma compreensão intuitiva e empática das ideias matemáticas de uma cultura, que é essencial para conduzir um trabalho de campo etnográfico eficaz por meio de uma abordagem dialógica da etnomodelagem.

Palavras-chave: Ethnomodelagem, Etnomatemática, Modelagem Matemática, Etnomodelos,

[1*] Doutor em Educação pela New Mexico State University (NMSU). Professor na Universidade Federal de Ouro Preto (UFOP), Ouro Preto, Minas Gerais, Brasil. Endereço para correspondência: Rua Pref. José de Castro, 31, Nossa Senhora de Lourdes, Ouro Preto, Minas Gerais, CEP: 35.404-510. E-mail: oreydc@ufop.edu.br.
[2**] Doutor em Educação pela California State University, Sacramento (CSUS). Professor na Universidade Federal de Ouro Preto (UFOP), Ouro Preto, Minas Gerais, Brasil. Endereço para correspondência: Rua Pref. José de Castro, 31, Nossa Senhora de Lourdes, Ouro Preto, Minas Gerais, CEP: 35.404-510. E-mail: milton.rosa@ufop.edu.br.

Abordagem Dialógica.

RESUMEN

La aplicación de técnicas etnomatemáticas junto con las herramientas de modelización nos permite ver una realidad diferente y nos da una visión de las matemáticas holísticas. En esta perspectiva, el enfoque pedagógico que conecta los aspectos culturales de las matemáticas con sus aspectos escolares y/o académicos se denomina etnomodelación, que es un proceso de traducción y elaboración de problemas y preguntas tomados de sistemas que son parte de la realidad de cualquier grupo cultural determinado. En este artículo hemos propuesto un objetivo alternativo para la investigación, que es la adquisición de conocimientos tanto émicos como éticos para la implementación de la etnomodelación. El conocimiento émico es esencial para una comprensión intuitiva y empática de las ideas matemáticas de una cultura, y es esencial para llevar a cabo un trabajo de campo etnográfico eficaz a través de un enfoque dialógico de la etnomodelación.

Palabras clave: Etnomodelación, Etnomatemática, Modelación Matemática, Etnomodelos, Abordaje Dialógico.

1 INTRODUCTION

When researchers investigate the knowledge possessed by members of distinct cultural groups, they may be able to find unique mathematical ideas and characteristics that we consider as ethnomathematics. However, an outsider’s understanding of cultural traits is always an interpretation that may emphasize its inessential features to the misinterpretation of this distinctly unique or culturally specific mathematical knowledge.

The challenge that arises from this approach is how the culturally bound mathematical ideas can be extracted or understood without letting the culture of the researcher or investigator interfere with the culture of the members of the cultural group under study. This may happen when the members of cultural groups have their own interpretation of their culture, which is called an *emic* perspective as opposed to an outsider’s interpretation, which is called the *etic* perspective.

Researchers, investigators, educators, and teachers who take on an emic perspective believe that many factors such as cultural and linguistic backgrounds, social, moral values, and lifestyle come into play when mathematical ideas, concepts, procedures, and practices are developed by the people of their own culture. Different cultural groups have developed different ways of doing mathematics in order to understand and comprehend their own cultural, social, political, economic, and natural environments (Rosa, 2010).

https://doi.org/10.63036/ReTEM.2965-9698.2025.v3.400 2965-9698

Furthermore, every cultural group has developed unique and often distinct ways to *mathematize* their own realities (D'Ambrosio, 1990). Mathematization is the process in which individuals from different cultural groups come up with different mathematical tools that can help them organize, analyze, comprehend, understand, model, and solve specific problems located in the context of their own real-life situation (Rosa & Orey, 2006).

These tools allow them to identify and describe specific mathematical ideas, concepts, procedures, or practices in a general context by schematizing, formulating, and visualizing a problem in different ways, discovering relations and regularities, and transferring a real-world problem to a mathematical idea through mathematization.

It is important to search for alternative methodological approaches as western-academic mathematical practices are accepted worldwide in order to record historical forms of mathematical ideas and concepts that occur in different cultural contexts. One alternative methodological approach to this is ethnomodelling, which is considered as a practical application of ethnomathematics and adds a cultural perspective to modelling concepts (Rosa & Orey, 2010a). When justifying the need for a culturally bound view on mathematical modelling, our sources are rooted on the theory of ethnomathematics developed in Brazil by Ubiratan D'Ambrosio.

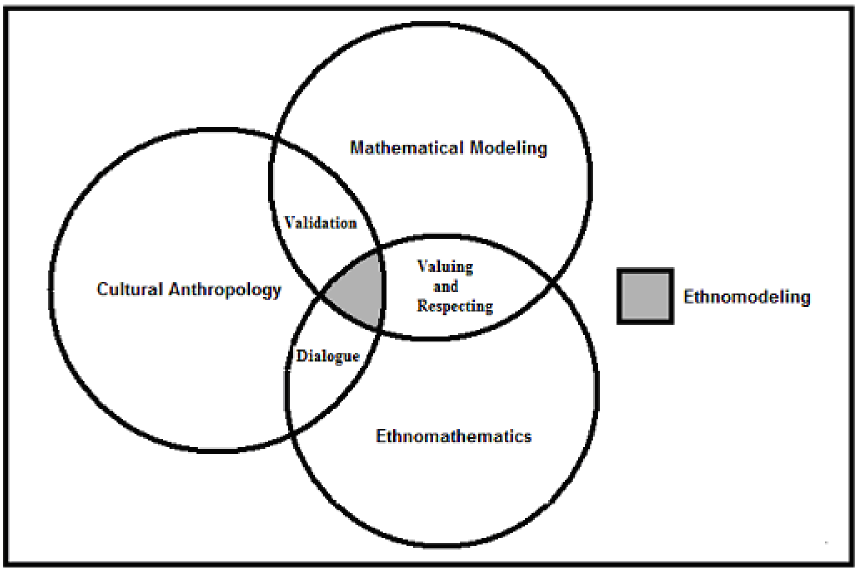
2 ETHNOMODELLING

Again, ethnomodelling is the study of mathematical ideas and procedures elaborated by members of distinct cultural groups. It involves the mathematical practices developed, used, practiced, and presented in diverse situations in the daily life of the members of these groups (Rosa & Orey, 2010a). This context is holistic and allows those engaged in this process to study mathematics as a system or an ecology taken from their own contextual reality in which there is an equal effort to create an understanding of all components of these systems as well as the interrelationship among them (D'Ambrosio, 1993; Bassanezi, 2002; Rosa & Orey, 2003).

Eglash, Bennett, O'Donnell, Jennings, and Cintorino (2006) affirmed that researchers and investigators such as Ascher (2002), Eglash (1999), Gerdes (1991), Orey (2000), Urton (1997), and Rosa and Orey (2009) "have revealed [in their research] sophisticated mathematical ideas and practices that include geometric principles in craft work, architectural concepts, and practices in the activities and artifacts of many indigenous, local, and vernacular cultures" (p. 347). These concepts are related to numeric relations found in measuring, calculation, games, divination, navigation, astronomy, modelling, and a wide variety of other mathematical procedures and cultural artifacts (Eglash at all, 2006).

According to Rosa and Orey (2010b) understand that "Using ethnomodelling as a tool towards pedagogical action of an ethnomathematics program, students have been shown to learn how to find and work with authentic situations and real-life problems" (p. 60). In this context, Rosa and Orey (2010a) stated that ethnomodelling may be considered as the intersection area of cultural anthropology, ethnomathematics, and mathematical modelling. Figure 1 shows ethnomodelling as intersection of three research fields: cultural anthropology, mathematical modelling, and ethnomathematics.

Figure 1: Ethnomodelling as an Intersection of Three Research Field



Source: Rosa and Orey (2010b)

In this regard, researchers such as Eglash at al. (2006) and Rosa and Orey (2006) use the term *translation* to describe the process of modelling local cultural systems, which may have a Western school and/or academic mathematical representation. On the other hand, Eglash at al. (2006) stated that often indigenous designs are merely analyzed from a Western view such as the applications of the symmetry classifications from crystallography to indigenous textile patterns. In this context, ethnomathematics also makes use of modelling by

https://doi.org/10.63036/ReTEM.2965-9698.2025.v3.400 2965-9698

attempting to use it in order to establish relations between the local conceptual framework and the mathematics embedded in relation to local designs.

Thus, Eglash at al. (2006) stated that in some cases, “the translation to Western mathematics is direct and simple such as counting systems and calendars” (p.347). However, there are cases in which mathematical ideas and concepts are “embedded in a process such as iteration in bead work, and in Eulerian paths in sand drawings” (p.348). In this regard, the translation applied in this process is best referred to as ethnomodelling because “the act of translation is more like mathematical modelling” (Eglash at al., 2006, p. 348).

Here, ethnomodelling stands in agreement with Eglash at al. (2006) who stated that “mathematics knowledge can be seen as arising from emic rather than etic origins” (p. 349). This seems reasonable as ethnomathematics often applies “modelling to establish relations between the indigenous conceptual framework and the mathematics embedded in related indigenous designs” (Eglash at al., 2006, p. 349).

Ethnomodelling aims to develop pedagogical actions whose epistemological assumptions are linked with a broad historiography that starts from the reality and background of the learners, and which seeks to value and respect their historical knowledge acquisition through a cognitive approach with an in-depth cultural foundation. This general view of mathematics from a cultural perspective is one of the main goals of ethnomodelling (Rosa & Orey, 2021).

In this context, the emphasis of ethnomodelling takes in consideration the essential processes found in the construction and development of scientific and mathematical knowledge, which includes often curious and unique aspects of collection, creativity, and invention. In so doing, it is impossible to imprison mathematical concepts in registers of univocal designation of reality because there are distinct systems that provide an unambiguous representation of reality as well as universal explanations (Craig, 1998).

This means that mathematics was not conceived as a universal language because its principles, concepts, and foundations were not the same everywhere (Rosa, 2010). Craig (1998) also stated that “the choice among equivalent systems of representation can only be founded on considerations of simplicity, for no other consideration can adjudicate between equivalent systems that univocally designate reality” (p. 540).

For example, Rosa and Orey (2006) argued that the processes of production of scientific and mathematical ideas, concepts, procedures, and practices operate in the register

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of the interpretative singularities regarding the possibilities for a symbolic construction of knowledge in different cultural groups.

3 THE EMIC (LOCAL) AND ETIC (GLOBAL) CONSTRUCTS OF ETHNOMODELLING

Ethnomodelling privileges the organization and presentation of mathematical ideas, concepts, procedures, and practices developed by the members of distinct cultural groups in order to facilitate its communication and transmission across generations. The elaboration of models that represent these systems are representations that help to understand and comprehend the world by using small units of information, named ethnomodels, which link cultural heritage with the development of mathematical practices (Rosa & Orey, 2010a).

This helps in the use of necessary information to solve problems taken from reality. It is our understanding that this approach may help the organization of the pedagogical action that occurs in classrooms through the use of the emic and etic aspects of this mathematical knowledge. Yet, it is necessary to emphasize that the "challenge for Intercultural Education is to establish and maintain the balance between conformity with its general guiding principles and the requirements of specific cultural contexts" (UNESCO, 2006, p. 9).

By using an ethnomodelling perspective in investigations or in pedagogical action, the emic (local) constructs are the accounts, descriptions, and analyses expressed in terms of the conceptual schemes and categories that are regarded as meaningful and appropriate by the members of the cultural group under study. This means that an emic construct is in accordance with the perceptions and understandings deemed appropriate by the insider's culture. The validation of the emic (local) knowledge comes with a matter of consensus, which is the consensus of local people who must agree that these constructs match the shared perceptions that portray the characteristic of their culture (Lett, 1996).

In other words, the emic approach tries to investigate mathematical phenomena and their interrelationships and structures through the eyes of the people native to a particular cultural group. It is paramount to note that the particular research technique used in acquiring emic mathematical knowledge has nothing to do with the nature of that knowledge. Lett (1996) affirmed that the "emic mathematical knowledge may be obtained either through elicitation or observation because it is possible that objective observers may infer local

https://doi.org/10.63036/ReTEM.2965-9698.2025.v3.400 2965-9698

perceptions” (p. 382) about mathematical ideas, concepts, techniques, procedures, and practices.

On the other hand, in ethnomodelling, the etic constructs are considered as accounts, descriptions, and analyses of mathematical ideas, concepts, procedures, and practices expressed in terms of the conceptual schemes and categories that are regarded as meaningful and appropriate by the community of scientific observers, researchers, and investigators (Lett, Orey: Rosa, 2021). An etic construct is precise, logical, comprehensive, replicable, falsifiable, and observer-researcher independent. In so doing, Lett (1996) states that the validation of the etic knowledge thus becomes a matter of logical and empirical analysis, in particular, the logical analysis of whether the construct meets the standards of falsifiability, comprehensiveness, and logical consistency, and then the empirical analysis of whether or not the mathematical concept has been falsified and or replicated.

Hence, Lett (1996) also emphasized that the particular research techniques that are used in the acquisition of etic (global) mathematical knowledge has no bearing on the nature of that knowledge. Etic knowledge may be obtained at times through elicitation as well as observation, because it is entirely possible that native informants possess scientifically valid knowledge. In fact, D’Ambrosio (1990) argued that researchers and investigators have to acknowledge and recognize that local people possess scientifically mathematically valid knowledge.

4 ETHNOMODELS

Through history, researchers and investigators have made extensive use of mathematical procedures ranging from statistical methods for the elucidation of patterns in behavior to the mathematical representations of the logic processes of indigenous conceptual systems. Mathematical modelling and its models have been considered by some researchers metaphorically as a tool and by others as a way to extend anthropological and archaeological reasoning.

On the other hand, other researchers have decried the use of mathematical, and in particular, statistical and quantitative modelling as fundamentally in opposition to a humanistic approach to understanding human behavior and knowledge that takes into account contingency and historical embeddedness and in turn, decries universality. We argue that traditional mathematical modelling does not fully take into account the implications of cultural aspects of local and social systems.

The cultural component in this process is critical because its accounts “emphasize the unity of culture, viewing culture as a coherent whole, a bundle of [mathematical] practices and values” (Pollak; Watkins, 1993, p. 490) that are incompatible with the rationality of the elaboration of traditional mathematical modelling process. However, in the context of mathematical forms of knowledge, what is meant by the cultural component varies widely and ranges from viewing mathematical practices as socially learned and transmitted to the members of cultural groups to mathematical practices viewed as made up of abstract symbolic systems with an internal logic giving a symbolic system its mathematical structure.

If the former is considered, then it is the process by which knowledge transmission takes place from one person to another, which is central to elucidating the role of culture in the development of mathematical knowledge (D’Ambrosio, 1993). If the latter is considered, then culture plays a far-reaching and constructive role with respect to mathematical practices that cannot be induced simply through observation of these practices (Eglash at al., 2006).

In this regard, when mathematical knowledge is developed by members of a specific cultural group consists of abstract symbol systems whose form is the consequence of a unique internal logic; then learners learn specific instances of and definite usages of the symbol system as well as what is derived from those instances forms a cognatically based understanding of the internal logic of mathematical symbology systems.

In contrast, mathematical knowledge consists of socially learned and transmitted mathematical practices when constructing ethnomodels of mathematical practices found in sociocultural systems (Rosa & Orey, 2010). In this regard, mathematical practices are socially learned and transmitted to the members of cultural groups (D’Ambrosio, 1993).

Thus, the cognitive aspects needed in this framework are primarily decision processes by which the members of cultural groups either accept or reject an ethnomodel as part of their own repertoire of mathematical knowledge. We believe that the conjunction of these two scenarios appears to be adequate to the depth needed to encompass the full range of cultural-mathematical phenomena.

In effect, there are two ways in which we recognize, represent and make sense of a mathematical phenomenon that impinges upon our sensory apparatus:

https://doi.org/10.63036/ReTEM.2965-9698.2025.v3.400 2965-9698

a) First, there is a level of cognition that we share, to varying degrees, with the members of our own and other cultural groups. This level would include cognitive models that we may elaborate on at a non-conscious level, which serves to provide an internal organization of external mathematical phenomena in order to provide the basis upon which a mathematical practice takes place.
b) Second, there is a culturally constructed representation of external mathematical phenomena that also provides its internal organization. However, this representation arises through the formulation of abstract and conceptual structures that provides forms and organizations for external phenomena. In other words, the cultural construct provides representations for constructed mathematical realities.

The implications of mathematical modelling regarding to systems present in the reality are models of cultural constructs, which are considered as symbolic systems organized by the internal logic of members of cultural groups. According to Eglash at al. (2006) and Rosa and Orey (2010b) models built without a first-hand sense for the world being modeled should be viewed with suspicion.

In this regard, researchers and investigators, if not blinded by their prior theory and ideology, should come out with an informed sense of distinctions that make a difference from the point of view of the mathematical knowledge of the work being modeled. In so doing, they should, in the end, be able to tell outsiders (etic) what matters to insiders (emic).

5 THE EMIC-ETIC DILEMMA IN ETHNOMODELLING RESEARCH AND PEDAGOGICAL ACTION: A DIALOGICAL APPROACH

The concepts of emic and etic were first introduced by the linguist Pike (1954) who drew upon an analogy with two linguistic terms. Phon*emic*, which are the sounds used in a particular language and phon*etic*, which are the general aspects of vocal sounds and sound production in language. In other words, all the possible sounds human beings can make constitute the phonetics of the language. However, when people actually speak a particular language, they do not hear all its possible sounds. In this regard, as modeled by linguists, not all sounds make a difference because they are locally significant. This means that they are the phonemics of that language.

If we make an analogy in regard to ethnomodelling, it is possible to state that its emic perspective is concerned about differences that make mathematical practices unique from an *insider's* point of view. We argue that emic ethnomodels are grounded in what matters in the mathematical world of those being modeled. Yet, many ethnomodels are etic in the sense that they are built on an outsider's view about the world being modeled. In this context, etic ethnomodels represent how the modeler thinks the world works through systems taken from reality while emic ethnomodels represent how people who live in such worlds think these systems work in their own reality (Orey; Rosa, 2021).

We also would like to point out that the etic perspective always plays an important role in ethnomodelling research, yet the emic perspective should be also taken in consideration in this process. In this perspective, the emic ethnomodels sharpen the question of what an agent-based model should include to serve practical goals in modelling research. Thus, according to this context, mathematical ideas and procedures are etic if they can be compared across cultures using common definitions and metrics while the focus of the analysis of these aspects are emic if the mathematical ideas, concepts, procedures, and practices are unique to a subset of cultures that are rooted on the diverse ways in which etic activities are carried out in a specific cultural setting.

Currently, the debate between emic-etic is one of the most intriguing questions in ethnomodelling research since researchers continue to deal with questions such as:

1. *Are there mathematical patterns that are identifiable and/or similar across cultures?*
2. *Is it better to focus on these patterns particularly arising from the culture under investigation?*

While emic and etic are often thought of as creating a conflicting dichotomy, Berry (1999) emphasized that Pike (1967) originally conceptualized them as complementary viewpoints. According to this context, rather than posing a dilemma, the use of both approaches can deepen our understanding of important issues in scientific research and investigations about ethnomodelling. A suggestion for dealing with this dilemma is to use a combined emic-etic approach, rather than simply applying emic or etic dimensions of one culture to other cultures (Berry, 1990). A combined emic-etic approach requires researchers to first attain emic knowledge about all of the cultures in the study. According to Berry (1990), this allows them to put aside their culture biases, and to become familiar with the relevant cultural differences in each setting.

Usually, in ethnomodelling research, an emic analysis focuses on a single culture and employs descriptive and qualitative methods to study a mathematical idea, concept, procedure, or practice of interest. Its focus is on the study within the cultural group context in which the researcher tries to develop research criteria relative to internal characteristics or logic of the cultural system (Rosa; Orey, 2021).

In this perspective, Lett (1996) stated that meaning is gained relative to the context and therefore not easily, or of at all transferable to other contextual settings. For example, it is not intended to compare the observed mathematical patterns in one setting with mathematical patterns in other settings. This means that the primary goal of an emic approach is a descriptive idiographic orientation of mathematical phenomena because it puts emphasis on the uniqueness of each mathematical idea, concept, procedure, or practice developed by the members of cultural groups.

Thus, if researchers and educators wish to highlight meanings of these generalizations in specific or emic ways, then they need to refer to more precisely specified mathematical events. In contrast, an etic analysis would be comparative, examining many distinct mathematical cultural practices by using standardized methods (Lett, 1996).

This means that the etic approach tries to identify lawful relationships and causal explanations valid across different cultures. Thus, if researchers and educators wish to make statements about universal or etic aspects of mathematical knowledge, these statements need to be phrased in abstract ways.

On the other hand, Pike (1954) affirmed that an etics approach may be a way of getting at emics of the members of cultural groups. In this regard, the etic perspective may be useful for penetrating, discovering, and elucidating emic systems that were developed by members of different cultural groups (Pike, 1954).

In so doing, while the traditional concepts of emic and etic are important points of view for understanding and comprehending cultural influences on mathematical modelling, we would like to propose a distinctively different view on ethnomathematics and modelling research, which is referred as a *dialogical approach*.

Thus, it is necessary to "provide equity in public and social life, and to educate citizens who are open to intercultural dialogue and tolerant of each other's ways of being and thinking" (UNESCO, 2006, p. 7).

In a dialogical approach, the etic perspective claims that the knowledge of any given cultural group will have no necessary priority over its competing emic claims. According to this point of view, Eglash et al (2006) stated that there is a necessity to depend "on acts of "translation" between emic and etic perspectives" (p. 347). The Tipi ethnomodel below is a prime example of this approach. In other words, the cultural specificity may be better understood with the background of communality and the universality of theories and methods and vice versa.

In this context, these insights must be verified or falsified with methods independent of the subjectivity of the observer and researcher in order to achieve a scientific character. In so doing, it is important to analyze the insights that have been acquired through subjective and culturally contextualized methods. The rationale behind the emic-etic dilemma is the argument that mathematical phenomena in their full complexity can only be understood within the context of the culture in which they occur.

6 AN ETHNOMODELLING EXAMPLE: THE DIALOGICAL ETHNOMODEL OF THE SIOUX TIPI

According to Orey (2000), "spatial geometry is inherent by the shape of the tipi, and it was used to symbolize the universe in which the Plains Peoples lived" (p. 241). The word *tipi* from the Sioux language refers to a conical skin tent or dwelling common among the prairie peoples. Figure 2 shows the Sioux Tipi tent.

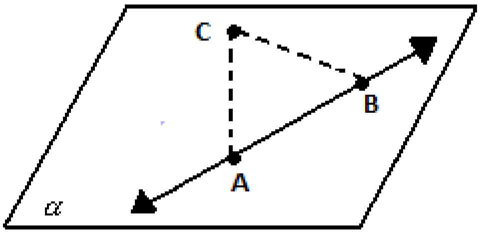
Figure 2: Sioux Tipi Watercolor on paper

Source: Karl Bodmer in 1833 from his travel to the U.S. 1832-1834

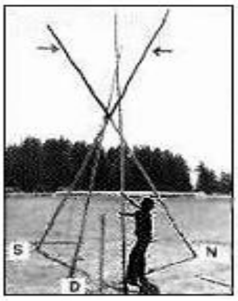
According to Orey (2000), the majority of Sioux people made use of the tripodal foundation because it is stronger and offers a firmer foundation than a quadripodal foundation. At best a quadripodal foundation may provide stable support for the Tipi with only three of its legs with the fourth leg providing no support for it. In this regard, Laubin and Laubin (1989) stated that the majority of tipi dwellers realized that the tripodal foundation was better in the defense against strong prevailing winds found on the open prairies of North America. There is evidence that the tripodal foundation is more common in areas in which the landscape has fewer obstacles and thus higher wind. Also for best support the tripodal foundation supporting the Tipi should form a near equilateral triangle.

In this context, an etic ethnomodel explains why a tripod is more flexible and stronger than a quadripodal structure. In this regard, imagine three points, A, B, and C, which are not collinear. There are an infinite number of planes that pass-through points A and B that contain the straight-line AB. Only one of these planes also passes through point C therefore we can say that three points are not collinear if they determine one plane.

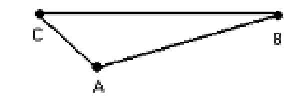
This means that these non-collinear points exist on one plane and that three collinear points determine the only plane. In other words, given any three non-collinear points, there is only one plane to which these same three points exist. We also can state that a plane can be determined by a straight line and a point that is not on the line.



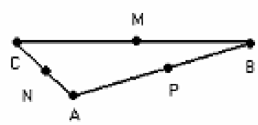
In terms of geometric concepts, this can be explained by using the plane postulate, which states that given any three non-collinear points, there is only one plane to which these same three points exist. This means that any three points define a plane completely and uniquely. Let us look at this information mathematically.



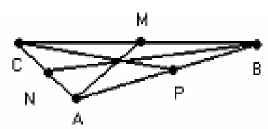
The base formed by the tripod is ΔABC.



The midpoints of each of the sides of ΔABC are points M, N, and P.



It is possible to match each vertex of ΔABC to the midpoint of each opposite sides that gives us the straight lines AM, BN, and CP.



These straight lines form three medians, which are the straight lines connecting the midpoint of each opposite side of the triangle and its vertex. The medians intersect at only one point called *centroid*. Archimedes demonstrated that medians of a triangle meet at its balance point or center of gravity, which is the centroid of the triangle (Orey, 2000). It is also important to highlight that Native Americans placed their fire and altar at this point in the tipi. In this regard, Orey (2000) stated that the “the center of the tipi holds a definite power and

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holiness” (p. 246). The Sioux people came to find this center because of its sacredness not just for necessity or aesthetics.

Through history, the nomadic prairie people observed that the tripodal foundation appeared to be perfectly adapted for the harsh environment in which they lived (Orey, 2000). In this regard, if we look at variation between tripodal and quadripodal use by the Sioux people, there is some evidence that they developed an understanding of geometry concepts such as triangles and their geometrical characteristics and properties. The reason Sioux people used a three-legged foundation rather than a four-legged foundation is that a tripodal foundation provides a stable support for the Tipi.

On the other hand, a quadripodal foundation is likely to have only three of its legs touching the ground while the fourth leg may slightly rise above the ground. When such a foundation is tilted to the side of the leg not touching the ground that leg may touch the ground, but some other leg may be raised. Thus the foundation with four legs tends to rock or move the Tipi as Sioux people are trying to protect themselves from the harsh and often windy conditions of the plains. This of course makes a quadripodal foundation unfit to provide a strong support for the Tipi.

According to this context, the tripodal foundation of the tipi has the advantage of providing a stabile structure for the Sioux people to live in an environment with strong winds and harsh weather. The mathematical idea implicit in this emic mathematical knowledge was passed to the members of the Sioux people across generations by the women of this tribal group, who were responsible for the construction, and upkeep of this unique conical dwellings. In this regard, D’Ambrosio (1993) affirmed that mathematical practices are socially learned and transmitted to the members of cultural groups.

In this example, an emic (local) observation sought to understand this mathematical practice for constructing the Tipi from the perspective of internal dynamics and relationships as influenced within that specific culture. On the other hand, an etic perspective provides a cross-cultural contrast and comparative perspectives by using some aspects of academic mathematics to translate this phenomenon for understanding for those from a different cultural background so as to comprehend and explain this mathematical practice as a whole from point of view of that from the outside. In so doing, the emic viewpoint clarifies intrinsic cultural distinctions while the etic perspective seeks objectivity as an outside observer across cultures (Pike, 1996).

In our opinion both perspectives are essential to understanding human behavior, especially mathematical characteristics. Thus, it is important to state that “One significant tension arises from the nature of Intercultural Education itself, which accommodates both *universalism* and *cultural pluralism"* (UNESCO, 2006, p. 9). This is the dialogical approach, which concerns the stability of relationships between these two different cultural approaches.

7 THE DIALOGICAL APPROACH INTO AN ETHNOMODELLING CURRICULUM

Mathematical knowledge of the members of cultural groups combined with Western-mathematical knowledge systems may result in a dialogical approach to mathematics education. An emic analysis of a mathematical phenomenon is based on internal structural or functional elements of a particular cultural group. An etic analysis is based on predetermined general concepts external to that cultural group (Lovelace, 1984).

Characteristics of culturally dialogical approaches in ethnomodelling research provide conditions regarding the development of intercultural competence, which is the “ability to communicate effectively in cross-cultural situations and to relate appropriately to a variety of cultural contexts” (Bennett & Bennett, 2004, p. 149).

Similarly, intercultural competence is the ability to “develop targeted knowledge, skills and attitudes that lead to visible behaviour and communication that are both effective and appropriate in intercultural interactions (Deardorff, 2006).

The emic perspective provides internal conceptions and perceptions of mathematical ideas and concepts while the etic perspective provides the framework for determining the effects of those beliefs on the development of the mathematical knowledge. In this perspective, the acquisition of mathematical knowledge is based on the applications of current mathematics curriculum, which may be assessed based on multiple instructional methodologies across various cultures (Rosa; Orey,2021).

In this regard, it could indeed be that one of the reasons for failure in many educational systems is that curriculum developers by using a *one size fits all* program, have ignored unique emic perspectives in school cultures. A dialectal approach includes the recognition of other epistemologies, and of holistic and integrated natures of mathematical knowledge of members of the diverse cultural groups found in many schools and urban centers. In this regard, an ethnomodelling curriculum provides an ideological basis for learning with and from the diverse cultural and linguistic elements presented by members of distinct cultural groups (Orey; Rosa, 2021).

In this kind of curriculum it would be crucial to understand that an etic construct is a mathematical-theoretical idea that is assumed to apply in all cultural groups while an emic construct is one that applies only to members of specific cultural groups. This means that there is concern for cultural bias occurring if educators and researchers assume that an emic construct is actually etic (Eglash at al., 2006). This results in an imposed etic perspective in which a culture-specific idea is wrongly imposed on the members of another culture.

An ethnomodelling curriculum that combines key elements of local knowledge with a dialogical approach is likely to produce students who can manage knowledge and information systems taken from their own reality and creatively apply this knowledge to other situations. This means that ethnomodelling can be considered part of a critical mathematics education because it is a learning process in which teachers encourage critical examination of multiple sources of knowledge and theories in diverse learning styles (Rosa; Orey, 2021).

In this approach, acquired knowledge is centered, located, oriented, and grounded on the cultural backgrounds of the students, which could be applied and translated appropriately by them and thus equip them to be fully productive locally and globally. According to Rosa and Orey (2010b), ethnomodelling is a pedagogical approach to reach this goal.

The nature of the previous mathematical knowledge of the students lends themselves to the principle of sequencing in curriculum development. By giving educators the freedom to start with previous mathematical knowledge and experience of their students, we can move from the familiar to the unfamiliar and from the concrete to the abstract in the process of promoting the acquisition of mathematical knowledge (Rosa & Orey, 2006). Also we can move from emic knowledge to etic knowledge and vice versa.

In this dialogical context, an ethnomodelling practice provides the underlying philosophy for knowledge generation and exchange within and between all subsystems of mathematics education. Key elements of an ethnomodelling curriculum approach ensure the balanced integration of the affective domain of educational objectives that are essential to the recognition and utilization of the students' previous knowledge.

8 FINAL CONSIDERATIONS

Today, numerous diverse indigenous mathematical knowledge systems and traditions are at risk of becoming extinct because of the rapidly changing natural and cultural environments and a fast pacing economic, social, environmental, political, and cultural changes occurring on a global scale. Many local mathematical practices disappear because of intrusion and imposition of foreign etic knowledge and technologies or from the development concepts that promise short-term gains or solutions to problems faced by a cultural group without considering their emic knowledge to solve these problems.

Not unlike the loss of global tropical rainforests, the tragedy of the impending disappearance of local knowledge is most obvious when a diversity of skills, technologies, cultural artifacts, problem solving strategies, and expertise are lost to all of us before being archived or understood.

Defined in that manner, the usefulness of the emic and etic distinction is evident. Like all human beings, researchers, educators, and teachers have been enculturated to some particular cultural worldview; they therefore need a means of distinguishing between the answers they derive as enculturated individuals and the answers they derive as observers.

Defining emics (local) and etics (global) in epistemological terms provides a reliable means of making that distinction. In this perspective, culture is a lens, shaping reality; it can be considered a blueprint, specifying a plan of action. At the same time, a culture is unique to a specific group of people. By utilizing the research provided by both approaches, we gain a more complete understanding of the cultural groups of interest.

In this article we have offered an alternative goal for research, which is the acquisition of both *emic (local)* and *etic (global)* knowledge for the implementation of ethnomodelling. Emic knowledge is essential for an intuitive and empathic understanding of mathematical ideas of a culture, and it is essential for conducting effective ethnographic fieldwork. Furthermore, emic knowledge is a valuable source of inspiration for etic hypotheses.

Etic knowledge (global), on the other hand, is essential for cross-cultural comparison, the essential components of ethnology, because such comparison necessarily demands standard units and categories. We also offered here a third perspective on ethnomodelling research, which is the *dialogical approach*; that makes use of both emic and etic knowledge and understandings through the processes of dialogue. Finally, we define ethnomodelling as the study of mathematical phenomena within a culture because it is a social construction and is culturally bound that adds the cultural aspects of mathematics to the modelling process.

REFERENCES

ASCHER, M. **Mathematics elsewhere**: an exploration of ideas across cultures. Princeton, NJ: Princeton University Press, 2002.

BASSANEZI, R. C. **Ensino-aprendizagem com modelagem matemática**. São Paulo, SP: Editora Contexto, 2002.

BENNETT, J.; BENNETT, M. Developing intercultural sensitivity: an integrative approach to global and domestic diversity. In: Landis, D.; J.; Bennett, M. (Eds.). **Handbook of intercultural training**. Thousand Oaks, CA: Sage, 2004. pp. 147-165.

BERRY, J. W. Imposed etics, emies and derived emics: their conceptual and operational status in cross-cultural psychologyIn: Headland, T. N.; Harris, M. (Eds). **Emics and etics**: the insider-outsider debate. Newbury Park, CA: Sage, 1990. pp. 84-89.

BERRY, J. W. Emics and etics: a symbiotic conception. **Culture & Psychology**, v. 5, p. 165-171, 1999.

CRAIG, E. **Routledge encyclopedia of philosophy**: questions to sociobiology. Volume 8. New York, NY: Routledge, 1998.

D'AMBROSIO, U. **Etnomatemática**: arte ou técnica de conhecer e aprender. São Paulo, SP: Editora Ática, 1990.

D'AMBROSIO, U. Etnomatemática: um programa. **A Educação Matemática em Revista**, v. 1, n.1, p. 5-11, 1993. Disponível em: https://www.sbembrasil.org.br/periodicos/index.php/emr/article/view/1662

DEARDORFF, D. K. The identification and assessment of intercultural competence as a student outcome of internationalization at institutions of higher education in the United States. **Journal of Studies in International Education**, v. 10, n. 3, p. 241-266, 2006. https://doi.org/10.1177/1028315306287002

EGLASH, R. **African fractals**: modern computing and indigenous design. New Brunswick, NJ: Rutgers University Press, 1999.

EGLASH, R., BENNETT, A., O'DONNELL, C., JENNINGS, S.; CINTORINO, M. Culturally situated designed tools: ethnocomputing from field site to classroom. **American Anthropologist**, v. 108, n. 2, p. 347-362, 2006.

LETT, J. Emic-Etic Distinctions. In: Levinson, D.; Ember, M. (Eds.). **Encyclopedia of cultural anthropology**. New York, NY: Henry Holt and Company, 1996. pp. 382-383

LOVELACE, G. Cultural beliefs and the management of agro-ecosystems. In: Rambo, T.; Sajise, P. E. (Eds.). **An introduction to human ecology research on agricultural systems in Southeast Asia**. Honolulu, HI: East–West Centre, 1984. pp. 194–205

MARTIN, J. N.; NAKAYAMA, T. K. **Intercultural communication in contexts**. Boston, MA: McGraw-Hill, 2007.

OREY, D. C. The ethnomathematics of the Sioux tipi and cone. In: Selin, H. (Ed.). **Mathematics across culture**: the history of non-western mathematics. Dordrecht, Netherlands: Kulwer Academic Publishers, 2000. pp. 239–252.

OREY, D. C.; ROSA, M. Ethnomodelling as a glocalization process of mathematical practices through cultural dynamism. **The Mathematics Enthusiast**, v. 3, n. 3, p. 438-468, 2021. https://doi.org/10.54870/1551-3440.1533

PIKE, K. L. **Emic and etic standpoints for the description of behaviour**. Glendale, IL: Summer Institute of Linguistics, 1954.

PIKE, K. L. **With heart and mind**: a personal synthesis of scholarship and devotion. Duncanville, TX: Adult Learning Systems, Inc, 1996.

POLLAK, R.; S. WATKINS. Cultural and economic approaches to fertility: proper marriage or mésalliance? **Population and Development Review**, v. 19, p. 467-496, 1993. https://doi.org/10.2307/2938463

ROSA, M. (2010). **A mixed-methods study to understand the perceptions of high school leader about English language learners (ELL)**: the case of mathematics. Doctorate dissertation. Educational Leadership. College of Education. Sacramento, CA: California State University, Sacramento.

ROSA, M.; OREY, D. C. (2003). Vinho e queijo: Etnomatemática e modelagem! **BOLEMA**, v. 16, n. 20, p. 1-16, 2003. Disponível em: https://www.periodicos.rc.biblioteca.unesp.br/index.php/bolema/article/view/10541

ROSA, M.; OREY, D. C. (2006). Abordagens atuais do programa etnomatemática: delinenando-se um caminho para a ação pedagógica. **BOLEMA**, v. 19, n. 26, p. 19-48, 2006. Disponível em: https://www.redalyc.org/pdf/2912/291221866003.pdf

ROSA, M.; OREY, D. Symmetrical freedom quilts: the ethnomathematics of ways of communication, liberation, and art. **Revista Latinoamericana de Etnomatemática**, v. 2, n. 2, p. 52-75, 2009. Disponível em: http://www.etnomatematica.org/v2-n2-agosto2009/rosa-orey.pdf

ROSA, M.; OREY, D. C. Ethnomodelling as a pedagogical tool for the ethnomathematics Program. **Revista Latinoamericana de Etnomatemática**, 3(2). 14- 23, 2010a. Disponível em: https://www.revista.etnomatematica.org/index.php/RevLatEm/article/view/25

ROSA, M,; OREY, D. C. Ethnomodelling: a pedagogical action for uncovering ethnomathematical practices. **Journal of Mathematical Modelling and Application**, v. 1, n. 3, p. 58-67, 2010b. Disponível em: http://proxy.furb.br/ojs/index.php/modelling/article/view/1972/1469

ROSA, M.; OREYD, D. C. An ethnomathematical perspective of STEM education in a glocalized world. **BOLEMA**, v. 35, n. 70, p. 840-876, 2021. https://doi.org/10.1590/1980-4415v35n70a14

UNESCO. **Literacy for life.** Education for all. EFA Global Monitoring Report. Paris, France: UNESCO, 2006. https://doi.org/10.54676/BQLU6071

URTON, G. **The social life of numbers**: a Quechua ontology of numbers and philosophy of arithmetic. Austin, TX: University of Texas Press, 1997.

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